Q31

$$\int \frac{1 + \cos 2x}{\sin^2 2x}\, dx$$

$$= \int \frac{1 + \cos 2x}{1 - \cos^2 2x}\, dx$$

$$= \int \frac{1 + \cos 2x}{(1 + \cos 2x)(1 - \cos 2x)}\, dx$$

$$= \int \frac{1}{1 - \cos 2x}\, dx$$

$$= \int \frac{1}{1 - 1 + 2\sin^2 x}\, dx$$

$$= \frac{1}{2} \int \csc^2 x\, dx$$

$$= -\frac{1}{2} \cot x + C$$

Alternate equivalent

$$-\frac{1}{2}(\csc 2x + \cot 2x)$$

$$= -\frac{1}{2}\left(\frac{1}{\sin 2x} + \frac{1}{\tan 2x} \right)$$

$$= -\frac{1}{2}\left(\frac{1 + \cos 2x}{\sin 2x} \right)$$

$$= -\frac{1}{2}\left(\frac{1 + 2\cos^2 x - 1}{2\sin x \cos x} \right)$$

$$= -\frac{1}{2}\left(\frac{\cos^2 x}{\sin x \cos x} \right)$$

$$= -\frac{1}{2} \cot x$$